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COMPRESSION SHOCKS IN TWO-DIMENSIONAL GAS FLOWS

By A. Busemann

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The following arguments on the compression shocks in gas flow start with a simplified representation of the results of the study made by Th. Meyer as published in the Forschungsheft 62 of the VDI, supplemented by several amplifications for the application.

In the treatment of compression shocks, the equation of energy, the equation of continuity, the momentum equation, the equation of state of the particular gas, as well as the condition of the second law of thermodynamics that no decrease of entropy is possible in an isolated system, must be taken into consideration. The result is that, in those cases where the sudden change of state according to the second law of thermodynamics is possible, there always occurs a compression of the gas which is uniquely determined by the other conditions.

First, it will be shown that the resulting relations can be easily expressed if the thermodynamic and the pure dynamic relations have been previously transformed so that pure thermodynamics, as well as pure dynamics, can be expressed simultaneously in one diagram. Since the static pressure p itself represents a state quantity as well as a dynamic quantity, one axis of the diagram may represent a p-axis. From the equation of energy for steady flow from a tank follows — the heat conduction being disregarded — the conventional relation that the kinetic energy of the unit mass $\frac{1}{2}w^2$ (w = velocity) is equal to the difference of the heat content of the tank i_0 and the momentary heat content i. Hence, when a w-axis is chosen as the other axis, the lines $w = $ Constant correspond to definite heat contents i and the diagram can be used as a distorted p,i diagram exactly like any other state diagram utilizing two state-quantities as axes. The following general relations in this diagram can be easily proved for adiabatic flow (fig. 1). For constant entropy, the negative differential quotient $-dp/dw$ represents the rate of flow ρw (ρ = gas density), as obtained from Bernoulli's equation: $\dfrac{-dp}{\rho} = \dfrac{1}{2}dw^2$.

The slope of the line of constant entropy accordingly represents for each point the rate of flow, that is, the reciprocal value of the surface necessary for the discharge of the unit mass. It immediately follows that the tangent to the entropy line on the p-axis cuts the momentum $p + \rho w^2$.

From these simple relations in the p,w diagram, the statement is
readily proved that normal compression shocks or compression shocks in
one-dimensional flow are possible only between those points which have a
common tangent on their entropy lines. Such states fulfill the equation
of state of the particular gas, because they are located on its p,i dia-
gram; they comply with the equation of energy, because the equation is
used to identify the w-axis; they satisfy continuity, because their entropy
lines have the same direction; and they have identical momentum, because
the tangents have equal intercepts on the p-axis (fig. 2).

The second law of thermodynamics contributes the fact that the later
one be the state of greater entropy. Since the cross section necessary
for the unit mass increases with the speed at supersonic velocity, and
hence the rate of flow decreases, the upward concave part of the entropy
lines signifies supersonic velocity, the upwardly convex part subsonic
velocity. Normal compression shocks have, therefore, supersonic speed
as initial state and subsonic speed as terminal state.

Extension of the arguments to include two-dimensional flow simply
involves the substitution of the w-axis for a u,v or velocity plane,
against which the pressures p are plotted, the surface of equal entropy
being obtained as surface of rotation of the entropy lines of the p,w dia-
gram (fig. 3). In isentropic flow, all states are situated on one single
surface of constant entropy. As stated elsewhere (reference 1), the gas
flow is unusually sensitive in cross sections in which a relatively
maximum rate of flow exists. In one-dimensional flow, the absolute
maximum rate of flow is through cross sections in which the flow velocity
equals the sonic velocity. In the p,w diagram, they are represented by
the point of inflexion of the entropy lines as the point of maximum slope
of the entropy line. In two-dimensional flow, all such cross sections are
normal to the directions of the curves of the main tangents on the saddle-
like curved region of the entropy surface. Then sensitive cross sections
with the relatively maximum rate of flow appear as steady sound waves in
two-dimensional supersonic flow, when minor disturbances (such as
roughening with a file) are applied at the boundary walls of the flow
(fig. 4).

The curves of the main tangents projected on the plane of the velocity
then give a network of lines by means of which the supersonic flow in the
prescribed channels can be pursued (fig. 5).

If the streamlines in a supersonic flow are deflected at a finite
angle toward the flow, say, by the boundary wall, for example, no stagnation
point need occur at this point like in the subsonic flow. The supersonic
flow can rather achieve the deflection by an oblique compression shock
(fig. 6), if the angle of deflection does not exceed a certain amount.
But this is accompanied by an entropy rise without which momentum, energy,
and continuity theorem cannot be fulfilled. The terminal states after a

compression shock are therefore no longer located on the surface of constant entropy, but within the pressure dome of constant initial entropy by reason of the entropy rise. On assuming the direction of the compression shock, or normal to it, the direction of the velocity variation, as given, it results in a p,w diagram above the particular straight line, in which the terminal state can be identified as the normal compression shock exactly like in figure 2 (fig. 7).

For a given velocity w_1 all terminal states after compression shocks lie on the tangential plane at the pressure dome in point p_1, $\underline{w}_1$ (fig. 8). In the tangential plane, the terminal states appear again as points of relatively maximum entropy on all rays through p_1, $\underline{w}_1$. In the projection on the velocity plane, the line connecting all terminal states $\underline{w}_2$ possible from $\underline{w}_1$ is termed shock polar. The shock polars give the possible deflections as well as the position of the shock surface perpendicular to the velocity difference $\underline{w}_1 - \underline{w}_2$ for each deflection. Figure 9 represents a shock polar diagram for air with $k = 1.405$, showing the shock polars from different starting points on the u-axis, along with the curves of constant entropy of the terminal state and indicated by the pressure ratio p'_o/p_o. By multiplication with p'_o/p_o it affords, for perfect gases, the height of the other surfaces of constant entropy from the heights of the initial adiabatic surface.

With these diagrams, it is possible to follow two-dimensional flows even in cases where compression shocks occur. For illustration, figure 10 shows a flat plate with a given angle of attack and figure 11 shows a symmetrical flow past a biconvex airfoil, and in figure 12, a schlieren record of real flow past such an airfoil. This example demonstrates that supersonic flows in which shocks occur, can also be treated graphically in close agreement with reality. Minor deflections may be treated by the methods of adiabatic flow.

Strong compression shocks present a certain difficulty if neighboring stream filaments pass through compression shocks of dissimilar intensity. Such flows are no longer irrotational and can then be treated approximately by the methods of potential flow only if the vortex strength is concentrated in certain stream lines. Each strip between two such stream lines can then be treated separately as potential flow and the bordering stream lines plotted in such a way that equal pressure and equal velocity direction appear in both adjacent strips. In the examples (figs. 10 and 11), the departure from potential flow was regarded as disappearingly small.

Discussion

Mr. Burgers, Delft, asked whether it was possible to draw a con-
clusion from the compression shocks about the wave resistance of bodies
at supersonic speeds.

Busemann: To compute the magnitude of wave resistance it is per-
missible for slender profiles to work with adiabatic compression shocks,
as given by Riemann (reference 2). The result is then invariably a
positive pressure on the surfaces which push the flow aside, and negative
pressure on the surfaces which contribute room to the flow. From this
the wave resistance (reference 3) follows immediately. The question of
where the work of resistance in the gas remains can be answered from the
compression shocks, even for slender profiles. As figure 11 indicates, com-
pression shocks are obtained, the strength of which abates simultaneously
with the disappearance of the wave field with increasing distance from
the profile. By integration with respect to all stream filaments, it
can be proved that the heating of the gas on traversing the compression
shocks precisely represents the work of resistance. The resistance momentum
follows likewise as momentum of the forward movement in the wake produced by
the shocks.

Translated by J. Vanier
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      Ermittlung von ebenen Strömungen mit Überschallgeschwindigkeit.
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2. Riemann — Weber: Partielle Differentialgleichung, 5th Edition, 1912,
      page 481.

3. Ackeret: Z. f. Flugtechnik u. Motorluftschiffahrt, 1925, page 72.
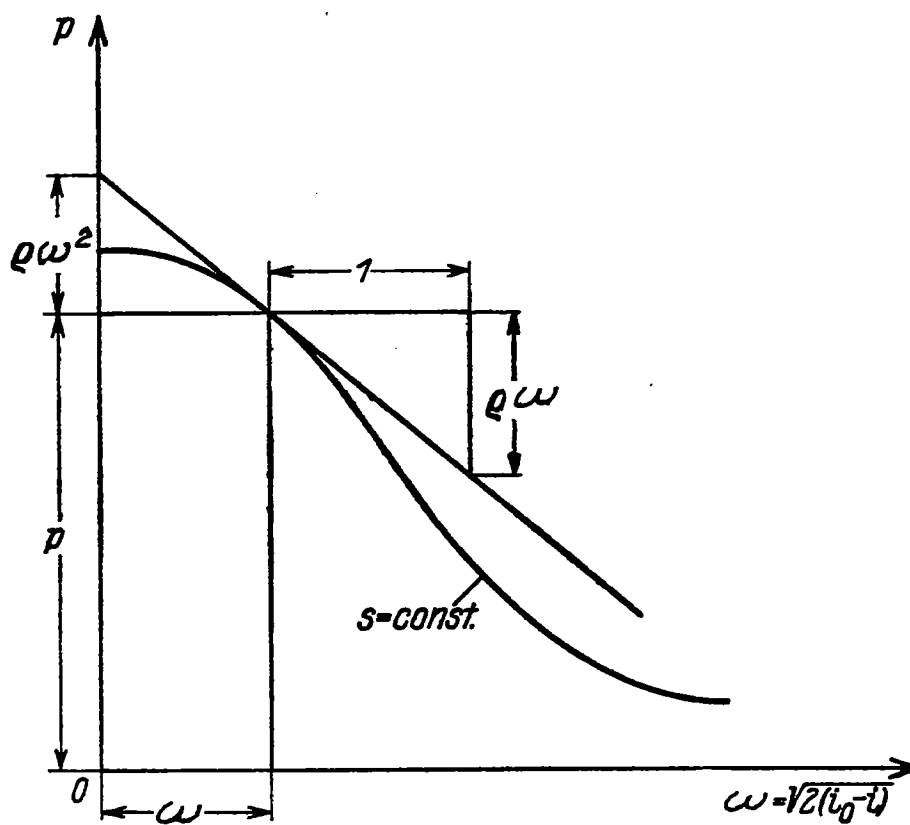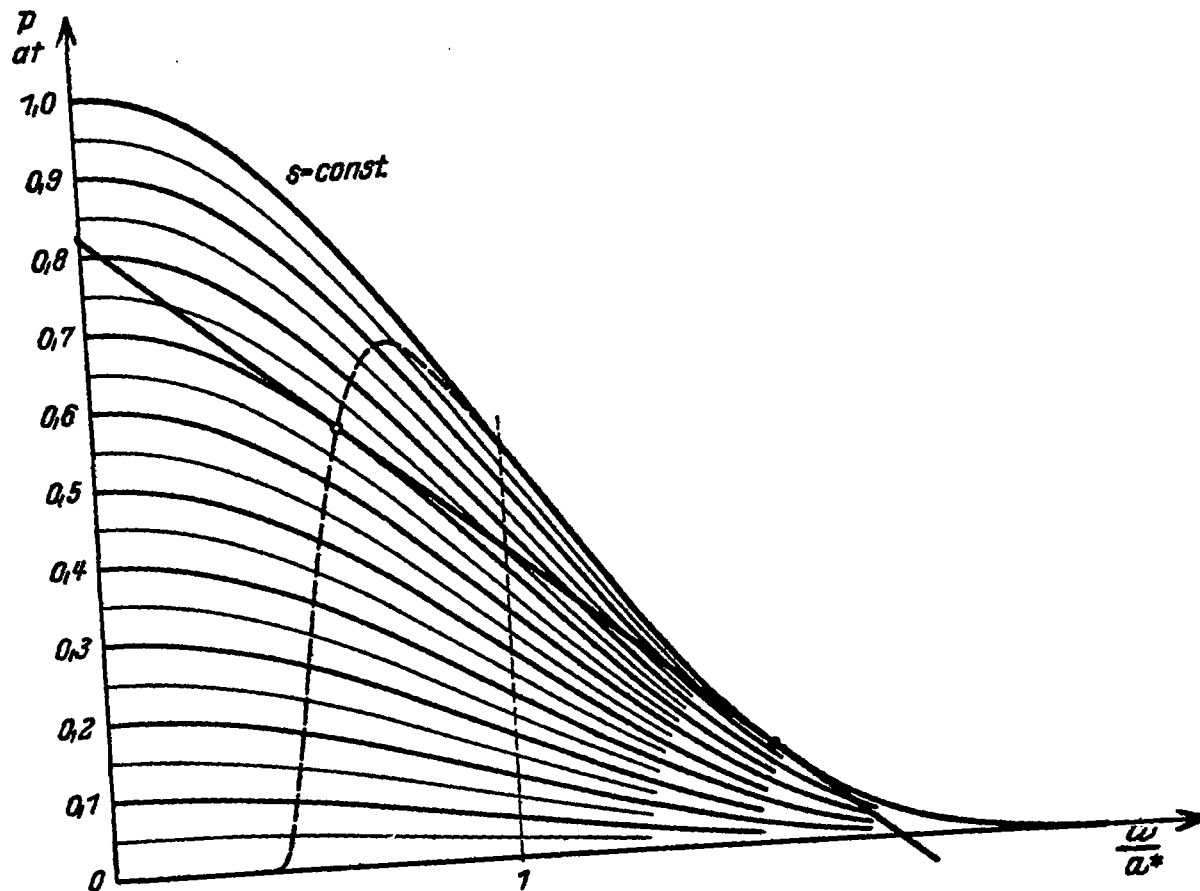
Figure 1.- Relations in the p,w diagram.
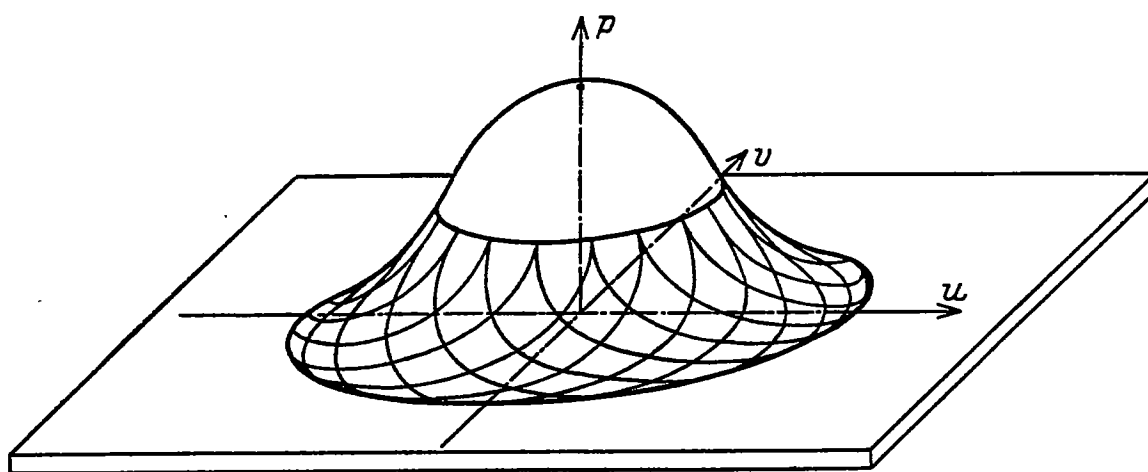
Figure 2.- Normal compression shock in the p,w diagram.

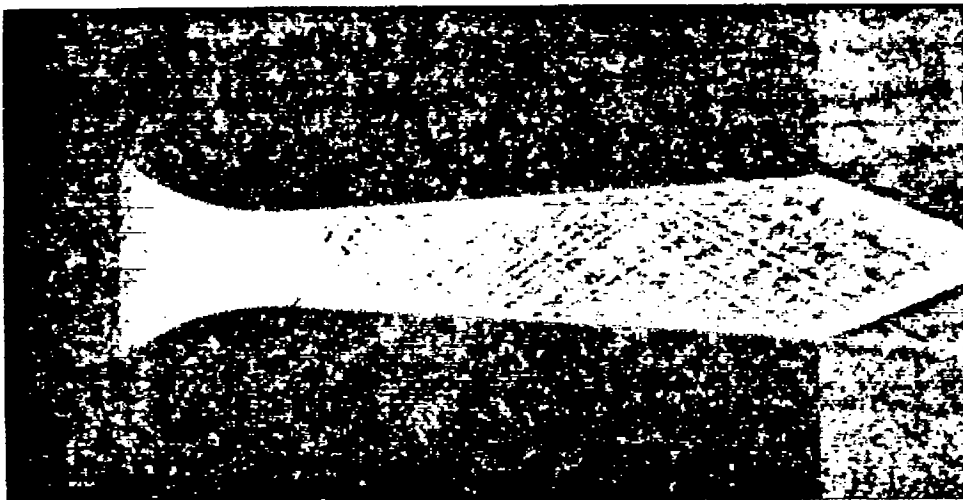Figure 3.-   p,u,v diagram for plane flow with constant entropy.

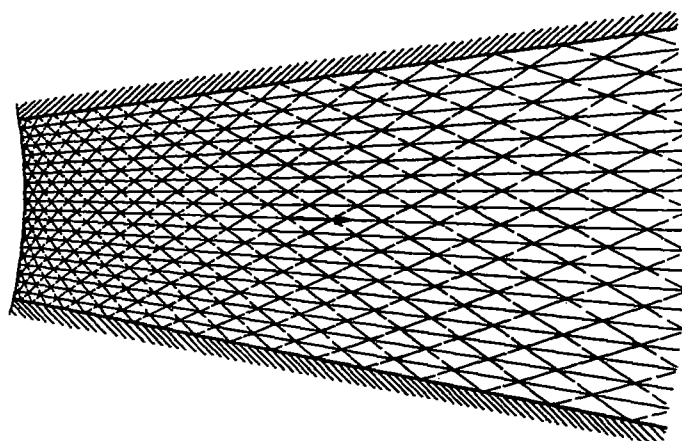Figure 4.- Schlieren photograph of steady sound waves.

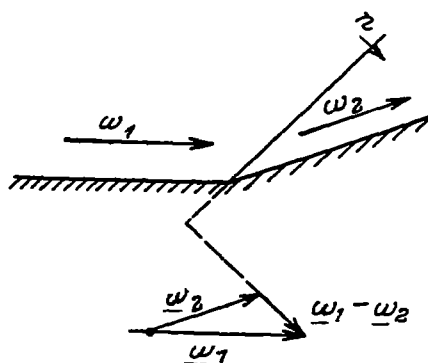Figure 5.- Graphical representation of flow of figure 4.

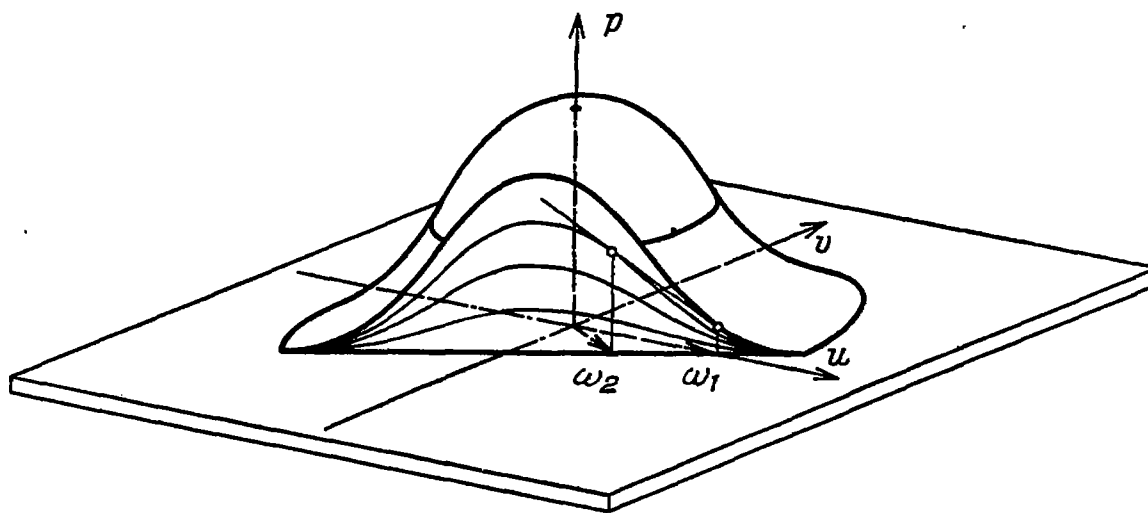Figure 6.- Compression shock at deflection by a finite angle.

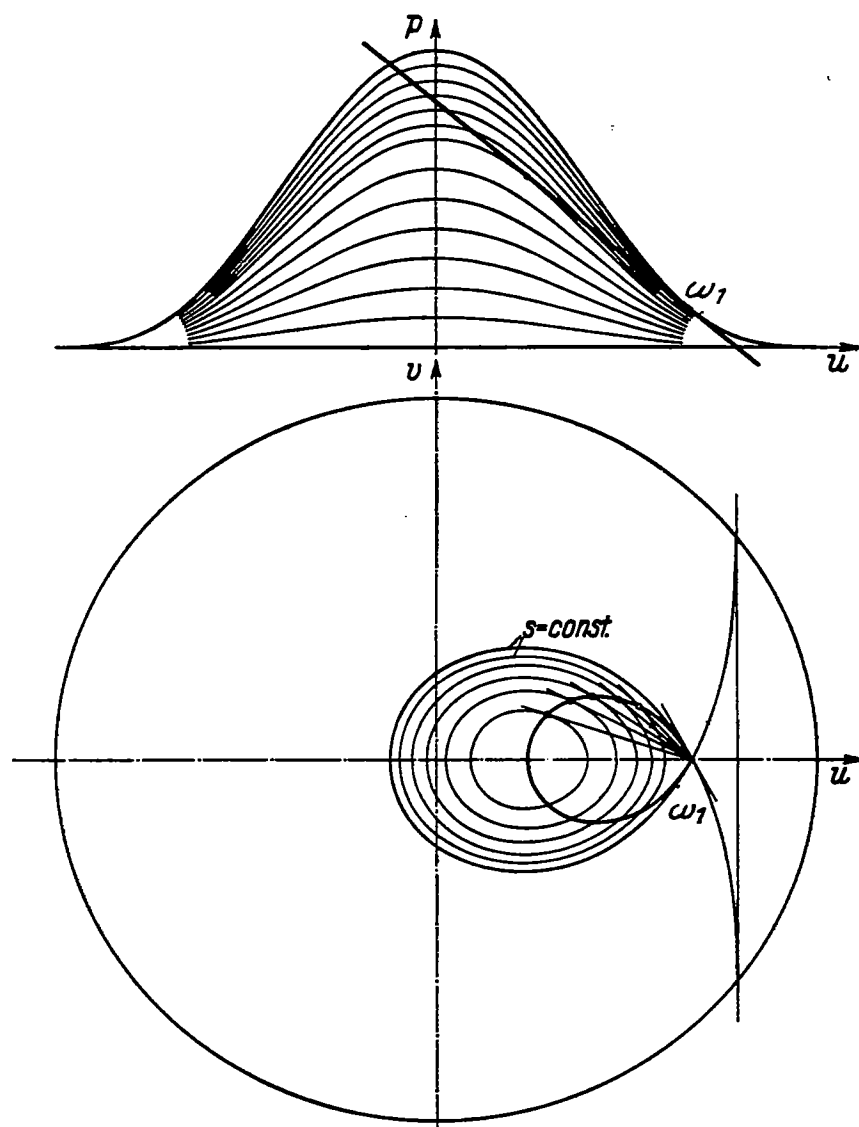Figure 7.- p,u,v diagram with entropy rise.

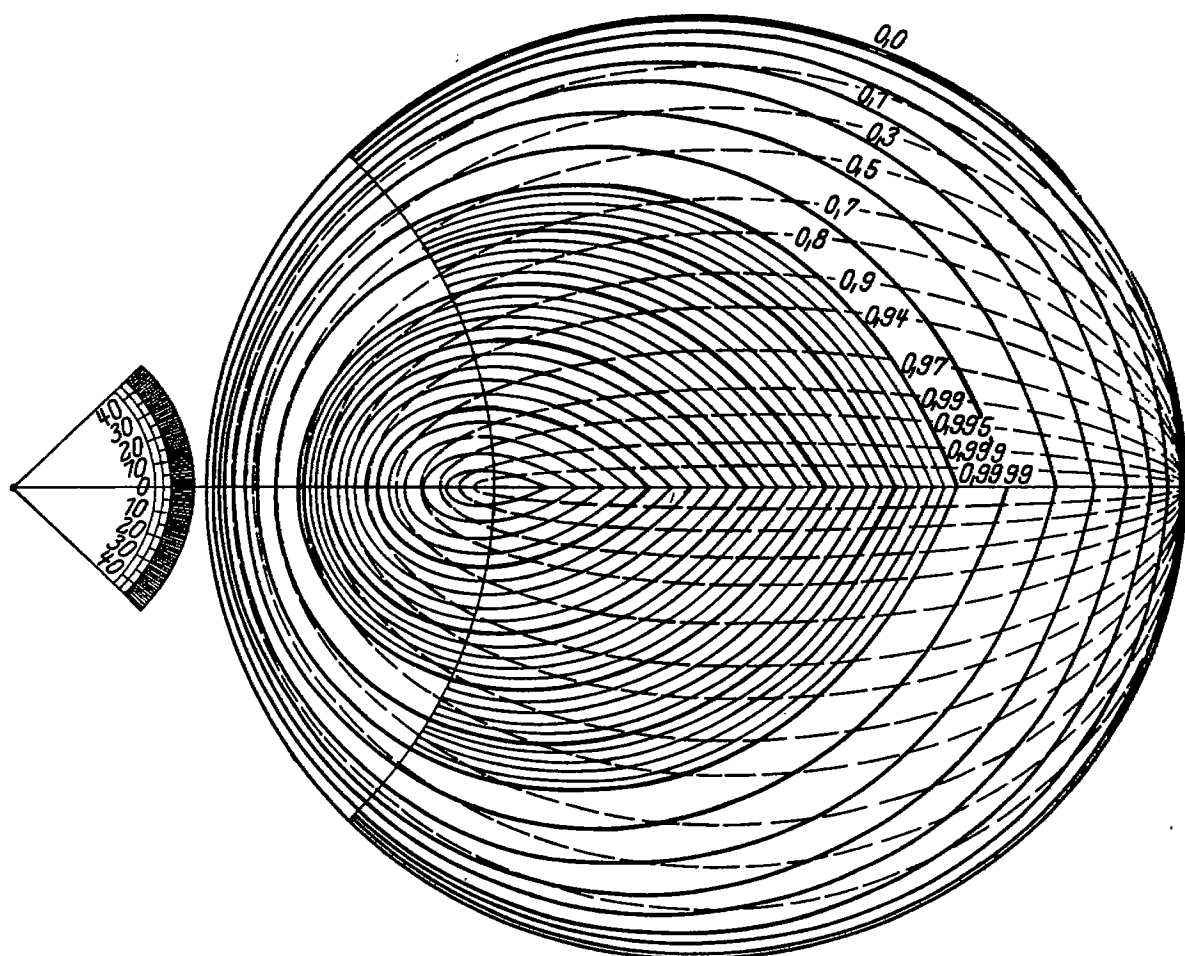Figure 8.- Shock polar in the tangential plane at the p dome.

Figure 9.- Shock polar diagram of air (k = 1.405). Shock polars, solid lines;
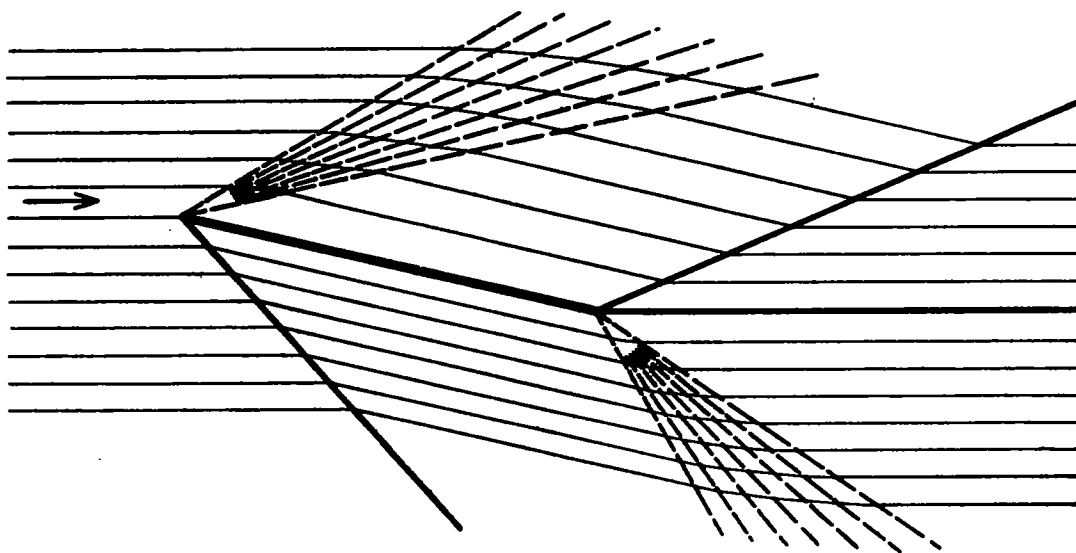$p_0{}'/p_0$ curves, broken lines.

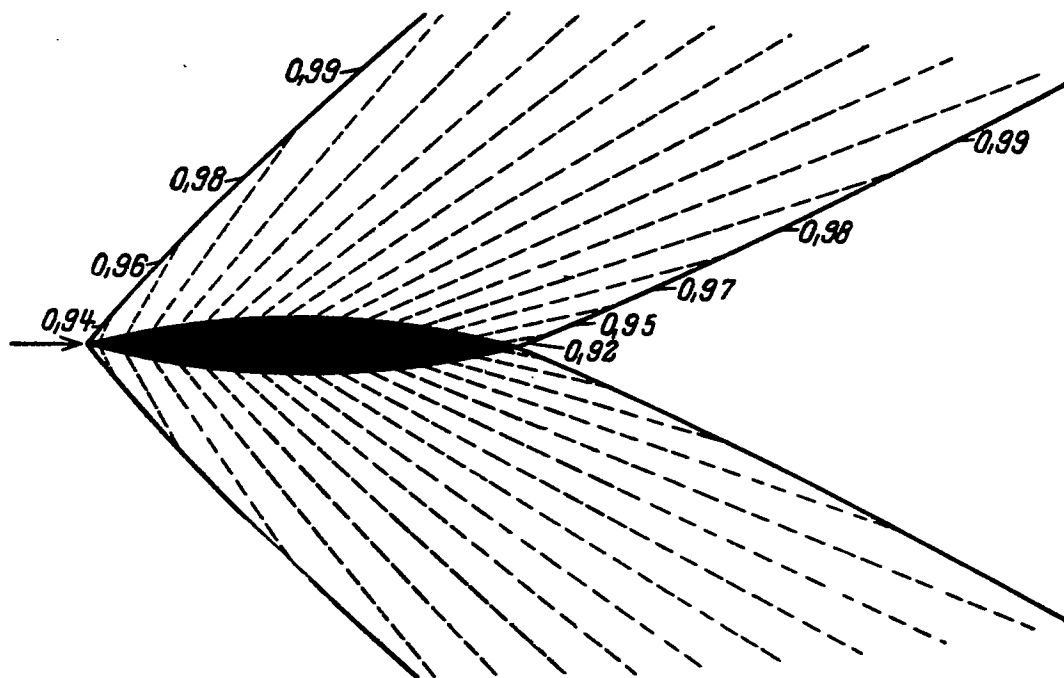Figure 10.- Flow past a flat plate with angle of attack.
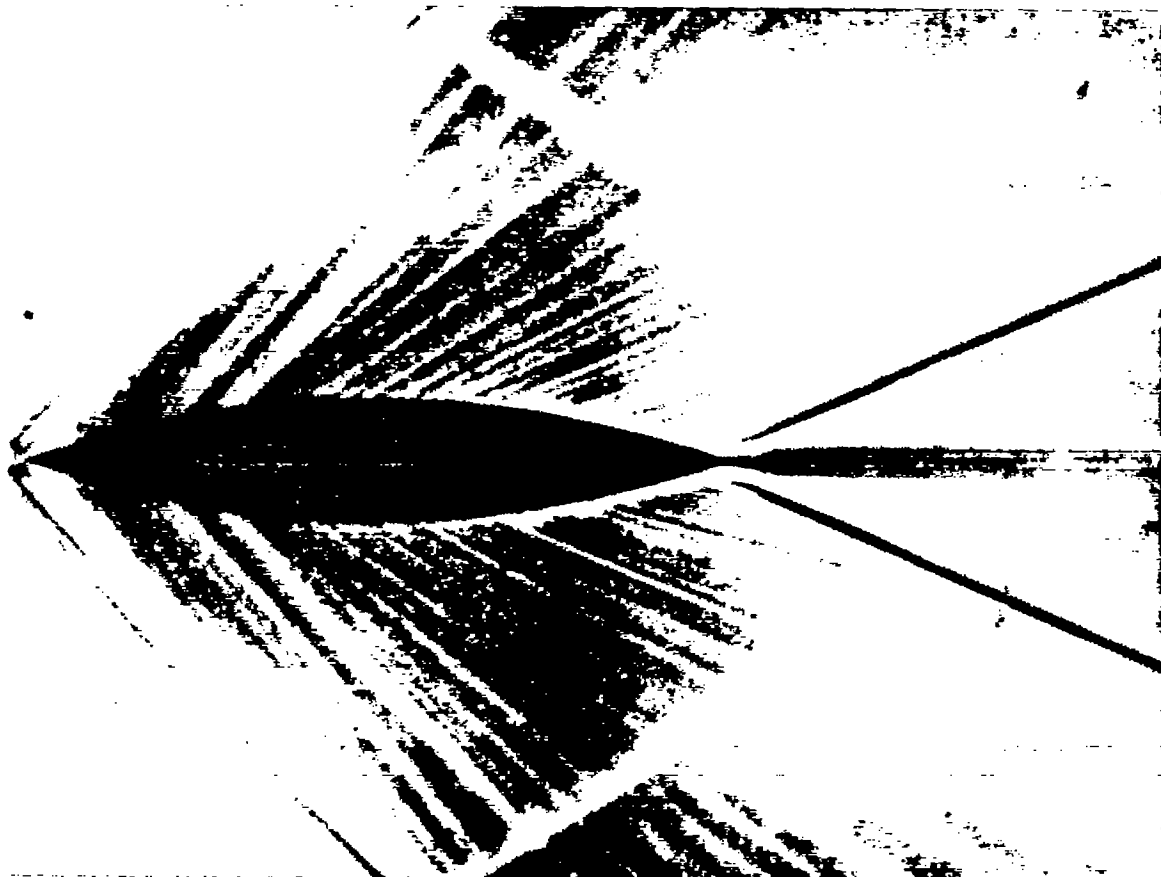
Figure 11.- Flow past a biconvex profile.

Figure 12.- Schlieren photograph of flow past a biconvex profile.